

Dependence of the forward-backward multiplicity correlation on acceptance and distance between windows in rapidity and azimuth

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Abstract

The forward-backward (FB) multiplicity correlations in two windows separated in rapidity and azimuth are analyzed in the framework of the model with independent identical emitters (strings). Along with the short-range contribution, originating from the correlation between multiplicities produced by a single emitter, the long-range contribution, originating from the fluctuation in the number of strings, is taken into account. The dependencies of the correlation coefficient on the rapidity and azimuthal acceptance of the windows and on the corresponding gaps between them are studied. It is shown that the analysis of these dependencies enables to separate the contributions of two above mechanisms. We see that the presence of the pair correlations between particles produced by one string turns the strings into non-poissonian emitters. The connection of the FB multiplicity correlation coefficient with the double inclusive cross section, the two-particle correlation function and the so-called untriggered di-hadron correlation is also traced. The suitable observables are proposed.

1 Introduction

For a long time, considerable attention is devoted to the experimental [1]-[6] and theoretical investigations of the forward-backward (FB) correlation - the correlation between multiplicities n_F and n_B of charged particles produced in two separated in rapidity windows (“forward” and “backward”) in high-energy pp and AA collisions (see [7]-[19] and references therein). The main problem in the analysis of this correlation is the separation of the so-called “volume” contribution, originating from the event-by-event fluctuation in the number of emitting sources.

In paper [17] it was suggested to use for this purpose the information on the event multiplicity in an additional third rapidity window, but as discussed in [18] it complicates the interpretation of obtained results. In present paper we argue that the investigation of the FB correlation between multiplicities in windows separated both in rapidity and in azimuth can enable not only to separate the volume contribution, originating from the fluctuation in the number of sources, but also to obtain the important quantitative physical information on the magnitude of this fluctuation in the processes under consideration.

We also show that the traditional definition of the FB correlation coefficient leads to its strong dependence on the acceptance of the windows, with the correlation coefficient going to zero with the acceptance. As consequence the results obtained for the windows of different width can’t be compared directly. In this connection we propose suitable observables for the FB correlation studies, which have some finite limit when the acceptance go to zero.

To check our observations we use the simple two stage model [9, 10, 20], inspired by a string picture of hadronic interactions. In this model one suggests that at the initial stage

of interaction some number N of strings are formed, which then are considered as identical independent emitters of observed charge particles. In previous note [21] we have considered only the long-range (LR) part of the correlation, originating from the fluctuation in the number of emitters (strings).

In the present paper we also take into account the short-range (SR) correlation between particles produced by a single string. This SR correlation can arise due to very different physical processes such as the details of string break up, the formation and decay of clusters, resonances or minijets during the string fragmentation. We show that the presence of such SR correlation along with influence on the FB multiplicity correlation turns the string into non-poissonian emitter.

The paper is organized as follows. In Sec. II we discuss the different version of the definition of the FB correlation coefficient and generalize these definition for the case of windows separated both in rapidity and azimuth. In Sec. III the connection of the FB correlation coefficient with two-particle correlation function is traced. On this base in Sec. IV we propose alternative suitable observables for the studies of the FB multiplicity correlations. In Sec. IV we shortly discuss the correspondence between the FB correlation and the so-called untriggered di-hadron correlation.

In Sec. V we go to the model calculations and introduce the pair correlation function of a single string. In Sec. VI in the framework of the model we find the resulting expression for the FB correlation coefficient. In Sec. VII we analyze the typical dependence of the FB correlation coefficient on the distances between windows in rapidity and azimuth and discuss the separation of the contributions of two mentioned mechanisms.

Appendix A describes the calculation of integrals over rapidity and azimuth windows. In Appendix B we present the alternative derivation of the basic formula for the FB correlation coefficient.

2 Definition of the FB correlation coefficient

Traditionally [1, 2, 4, 5] the FB correlation coefficient is defined as a coefficient b in linear regression

$$\langle n_B \rangle_{n_F} = a + b n_F . \quad (1)$$

In this case

$$b = \frac{\langle n_F n_B \rangle - \langle n_F \rangle \langle n_B \rangle}{D_{n_F}} , \quad (2)$$

where D_{n_F} is the variance of the multiplicity in the forward window

$$D_{n_F} = \langle n_F^2 \rangle - \langle n_F \rangle^2 . \quad (3)$$

Clear that the value of such defined correlation coefficient changes, if one will change independently the acceptances of the forward and/or backward windows. To avoid this trivial influence one can go from n_F and n_B to the relative or scaled observables [22] $\nu_F = n_F / \langle n_F \rangle$ and $\nu_B = n_B / \langle n_B \rangle$. In these observables $\langle \nu_B \rangle_{\nu_F} = a_{rel} + b_{rel} \nu_F$ and

$$b_{rel} = \frac{\langle \nu_F \nu_B \rangle - 1}{\langle \nu_F^2 \rangle - 1} = \frac{\langle n_F \rangle}{\langle n_B \rangle} b , \quad (4)$$

In some papers [3, 6] the following symmetrized form of (2) is also used

$$b_{sym} = \frac{\langle n_F n_B \rangle - \langle n_F \rangle \langle n_B \rangle}{\sqrt{D_{n_F} D_{n_B}}} , \quad (5)$$

for which one can prove that $|b_{sym}| \leq 1$. Note that in the case of symmetric windows, when $\langle n_F \rangle = \langle n_B \rangle$ and $D_{n_F} = D_{n_B}$, all these definitions lead to the same result

$$b_{rel} = b_{sym} = b . \quad (6)$$

In present paper we study the correlation between multiplicities n_F and n_B in windows separated both in rapidity and in azimuth. Denote by δy_F , $\delta \varphi_F$ and δy_B , $\delta \varphi_B$ the width of the forward and backward windows in rapidity and in azimuth, and by y_F , φ_F and y_B , φ_B - the positions of the centers of the windows. We'll also use the following short notation for the acceptance of forward and backward windows

$$\delta_F \equiv \delta y_F \delta \varphi_F / 2\pi , \quad \delta_B \equiv \delta y_B \delta \varphi_B / 2\pi . \quad (7)$$

By

$$y_{FB} \equiv y_F - y_B , \quad \varphi_{FB} \equiv \varphi_F - \varphi_B \quad (8)$$

we denote the distance between the centers of the windows in rapidity and in azimuth.

These variables are simply connected with the gaps y_{gap} and φ_{gap} between window in rapidity and in azimuth, we'll not use the last variables, but note that:

$$y_{FB} = \frac{\delta y_F}{2} + y_{gap} + \frac{\delta y_B}{2} , \quad \varphi_{FB} = \frac{\delta \varphi_F}{2} + \varphi_{gap} + \frac{\delta \varphi_B}{2} , \quad (9)$$

or for symmetric windows, when $\delta y_F = \delta y_B = \delta y$ and $\delta \varphi_F = \delta \varphi_B = \delta \varphi$,

$$y_{FB} = y_{gap} + \delta y , \quad \varphi_{FB} = \varphi_{gap} + \delta \varphi . \quad (10)$$

3 Connection with two-particle correlation function

One can express the FB correlation coefficient through the two-particle correlation function $C_2(y_1, y_2; \varphi_1 - \varphi_2)$. For this, we have to introduce the $\rho_1(y, \varphi)$ and $\rho_2(y_1, \varphi_1; y_2, \varphi_2)$ - the one- and two-particle densities of charge particles:

$$\rho_1(y, \varphi) = \frac{d^2 N}{dy d\varphi} , \quad \rho_2(y_1, \varphi_1; y_2, \varphi_2) = \frac{d^4 N}{dy_1 d\varphi_1 dy_2 d\varphi_2} . \quad (11)$$

Then for the forward acceptance interval, at $y \in \delta y_F$ and $\varphi \in \delta \varphi_F$, we have [23]:

$$\int_{\delta y_F \delta \varphi_F} dy d\varphi \rho_1(y, \varphi) = \langle n_F \rangle , \quad (12)$$

$$\int_{\delta y_F \delta \varphi_F} dy_1 d\varphi_1 \int_{\delta y_F \delta \varphi_F} dy_2 d\varphi_2 \rho_2(y_1, \varphi_1; y_2, \varphi_2) = \langle n_F (n_F - 1) \rangle$$

and the same for a backward window $\delta y_B \delta \varphi_B$. Meanwhile at $y_1 \in \delta y_F$, $\varphi_1 \in \delta \varphi_F$ and $y_2 \in \delta y_B$, $\varphi_2 \in \delta \varphi_B$ we have

$$\int_{\delta y_F \delta \varphi_F} dy_1 d\varphi_1 \int_{\delta y_B \delta \varphi_B} dy_2 d\varphi_2 \rho_2(y_1, \varphi_1; y_2, \varphi_2) = \langle n_F n_B \rangle . \quad (13)$$

Recall that $\langle n_F \rangle$ is an average multiplicity produced in the acceptance $\delta y_F \delta \varphi_F$.

The formulae (12) and (13) are the base for the experimental measurement of the one- and two-particle densities of charge particles $\rho_1(y, \varphi)$ and $\rho_2(y_1, \varphi_1; y_2, \varphi_2)$. For windows of small acceptance in rapidity and azimuth we have

$$\rho_1(y_F, \varphi_F) = \frac{\langle n_F \rangle}{\delta y_F \delta \varphi_F} , \quad \rho_2(y_F, \varphi_F; y_B, \varphi_B) = \frac{\langle n_F n_B \rangle}{\delta y_F \delta \varphi_F \delta y_B \delta \varphi_B} , \quad (14)$$

$$\rho_2(y_F, \varphi_F; y_F, \varphi_F) = \frac{\langle n_F(n_F - 1) \rangle}{(\delta y_F \delta \varphi_F)^2} . \quad (15)$$

Due to the rotation invariance in azimuth one has

$$\rho_1(y, \varphi) = \rho_1(y)/2\pi , \quad \rho_2(y_1, \varphi_1; y_2, \varphi_2) = \rho_2(y_1, y_2; \varphi_1 - \varphi_2)/(2\pi)^2 . \quad (16)$$

Then we introduce two-particle correlation function $C_2(y_1, y_2; \varphi_1 - \varphi_2)$ by a standard way:

$$C_2(y_1, y_2; \varphi_1 - \varphi_2) = \frac{\rho_2(y_1, y_2; \varphi_1 - \varphi_2)}{\rho_1(y_1)\rho_1(y_2)} - 1 . \quad (17)$$

By (12)–(17) we have

$$\langle n_F n_B \rangle - \langle n_F \rangle \langle n_B \rangle = I_{FB} , \quad (18)$$

$$D_{n_F} = \langle n_F \rangle + I_{FF} , \quad (19)$$

where

$$\langle n_F \rangle = \frac{\delta \varphi_F}{2\pi} \int_{\delta y_F} dy \rho_1(y) , \quad (20)$$

$$I_{FB} = (2\pi)^{-2} \int_{\delta y_F \delta \varphi_F} dy_1 d\varphi_1 \int_{\delta y_B \delta \varphi_B} dy_2 d\varphi_2 \rho_1(y_1) \rho_1(y_2) C_2(y_1, y_2; \varphi_1 - \varphi_2) , \quad (21)$$

$$I_{FF} = (2\pi)^{-2} \int_{\delta y_F \delta \varphi_F} dy_1 d\varphi_1 \int_{\delta y_F \delta \varphi_F} dy_2 d\varphi_2 \rho_1(y_1) \rho_1(y_2) C_2(y_1, y_2; \varphi_1 - \varphi_2) . \quad (22)$$

What gives for the correlation coefficient

$$b = \frac{I_{FB}}{D_{n_F}} = \frac{I_{FB}}{\langle n_F \rangle + I_{FF}} , \quad (23)$$

Further simplification of the integrals is discussed in Appendix A.

For example, in the case of FB windows separated only in rapidity (i.e. when $\delta \varphi_F = \delta \varphi_B = 2\pi$), by (105) we have

$$\langle n_F \rangle = \int_{\delta y_F} dy \rho_1(y) , \quad (24)$$

$$I_{FB} = \int_{\delta y_B} dy_1 \int_{\delta y_F} dy_2 \rho_1(y_1) \rho_1(y_2) C_2(y_1, y_2) , \quad (25)$$

$$I_{FF} = \int_{\delta y_F} dy_1 \int_{\delta y_F} dy_2 \rho_1(y_1) \rho_1(y_2) C_2(y_1, y_2) , \quad (26)$$

where

$$C_2(y_1, y_2) = \frac{1}{\pi} \int_0^\pi d\varphi C_2(y_1, y_2; \varphi) \quad (27)$$

and we used that

$$C_2(y_1, y_2; -\varphi) = C_2(y_1, y_2; \varphi) , \quad C_2(y_1, y_2; \varphi + 2\pi k) = C_2(y_1, y_2; \varphi) . \quad (28)$$

Note that if $\rho_2(y_1, \varphi_1; y_2, \varphi_2) = \rho_1(y_1, \varphi_1)\rho_1(y_2, \varphi_2)$, then there is no correlation: $C_2 = 0$, $I_{FB} = 0$, $I_{FF} = 0$ and by (19) $D_{n_F} = \langle n_F \rangle$ (see [23]).

For windows, which are small both in rapidity and in azimuth, and within which one can consider $C_2(y_1, y_2; \varphi_1 - \varphi_2)$ and $\rho_1(y)$ to be constant, we have

$$\langle n_F \rangle = \rho_1(y_F)\delta_F , \quad \langle n_B \rangle = \rho_1(y_B)\delta_B , \quad (29)$$

$$I_{FB} = \langle n_F \rangle \langle n_B \rangle C_2(y_F, y_B; \varphi_{FB}) , \quad (30)$$

$$I_{FF} = \langle n_F \rangle^2 C_2(y_F, y_F; 0) , \quad (31)$$

$$D_{n_F} = \langle n_F \rangle [1 + \langle n_F \rangle C_2(y_F, y_F; 0)] , \quad (32)$$

and

$$b_{rel} = \frac{\langle n_F \rangle}{\langle n_B \rangle} b = \frac{\langle n_F \rangle C_2(y_F, y_B; \varphi_{FB})}{1 + \langle n_F \rangle C_2(y_F, y_F; 0)} . \quad (33)$$

Recall our short notations (7) and (8).

We see that the correlation coefficient (4), defined in scaled variables, still depends through $\langle n_F \rangle$ on the acceptance δ_F of the forward window, that was observed earlier [20, 21] in a framework of a simple model.

In the case when both small FB windows are situated in the central region, where one can suppose the translation invariance in rapidity:

$$\rho_1(y) = \rho_0 , \quad C_2(y_1, y_2; \varphi) = C_2(y_1 - y_2; \varphi) \quad (34)$$

the formulae (29)–(33) admit further simplification:

$$\langle n_F \rangle = \rho_0 \delta_F , \quad \langle n_B \rangle = \rho_0 \delta_B , \quad (35)$$

$$D_{n_F} = \langle n_F \rangle [1 + \delta_F \rho_0 C_2(0; 0)] , \quad (36)$$

$$b_{rel} = \frac{\delta_F}{\delta_B} b = \frac{\delta_F \rho_0 C_2(y_{FB}; \varphi_{FB})}{1 + \delta_F \rho_0 C_2(0; 0)} . \quad (37)$$

At last for large windows situated in the central rapidity region along with (34) and (35) we must to use the formulae (19) and (23), with the following expressions for I_{FB} and I_{FF} :

$$I_{FB} = \rho_0^2 (2\pi)^{-2} \int_{\delta y_F \delta \varphi_F} dy_1 d\varphi_1 \int_{\delta y_B \delta \varphi_B} dy_2 d\varphi_2 C_2(y_1 - y_2; \varphi_1 - \varphi_2) , \quad (38)$$

$$I_{FF} = \rho_0^2 (2\pi)^{-2} \int_{\delta y_F \delta \varphi_F} dy_1 d\varphi_1 \int_{\delta y_F \delta \varphi_F} dy_2 d\varphi_2 C_2(y_1 - y_2; \varphi_1 - \varphi_2) . \quad (39)$$

The simplifications of the integrals in different cases are presented in Appendix A.

4 Alternative observables

From (29)–(33) we see that for small symmetric windows at $\delta_F = \delta_B \rightarrow 0$ we have $b \rightarrow 0$. This unpleasant dependence of the correlation coefficient on the width of the windows arises due to behavior of the variance D_{n_F} in the denominator of (23). Really by (30) and (32) we see that in this limit $I_{FB} \sim \delta_F \delta_B$ and $D_{n_F} \sim \delta_F$. We can rid of this drawback if we normalize the correlator $\langle n_F n_B \rangle - \langle n_F \rangle \langle n_B \rangle$ by the product $\langle n_F \rangle \langle n_B \rangle$ and introduce the observable

$$b_{mod} \equiv \frac{\langle n_F n_B \rangle - \langle n_F \rangle \langle n_B \rangle}{\langle n_F \rangle \langle n_B \rangle} = \left\langle \frac{n_F}{\langle n_F \rangle} \frac{n_B}{\langle n_B \rangle} \right\rangle - 1 . \quad (40)$$

Then for windows, which are small both in rapidity and in azimuth, by (30) we have

$$b_{mod} = C_2(y_F, y_B; \varphi_{FB}) \quad (41)$$

or in the case of the FB windows, which are small only in rapidity and large ($\delta\varphi_F = \delta\varphi_B = 2\pi$) in azimuth

$$b_{mod} = C_2(y_F, y_B) = \frac{1}{\pi} \int_0^\pi d\varphi C_2(y_F, y_B; \varphi) , \quad (42)$$

where we have take into account (24)–(27). We see that in contrast with b the b_{mod} has a finite limit at small acceptances of windows.

Another possibility, as it follows from (19), is to use for the normalization instead of D_{n_F} and D_{n_B} the differences $D_{n_F} - \langle n_F \rangle$ and $D_{n_B} - \langle n_B \rangle$ and to introduce

$$b_{rob} \equiv \frac{\langle n_F n_B \rangle - \langle n_F \rangle \langle n_B \rangle}{\sqrt{D_{n_F} - \langle n_F \rangle} \sqrt{D_{n_B} - \langle n_B \rangle}} . \quad (43)$$

Then again for windows, which are small both in rapidity and in azimuth, by (30) and (32) we have

$$b_{rob} = \frac{C_2(y_F, y_B; \varphi_{FB})}{\sqrt{C_2(y_F, y_F; 0) C_2(y_B, y_B; 0)}} \quad (44)$$

or

$$b_{rob} = \frac{C_2(y_{FB}; \varphi_{FB})}{C_2(0; 0)} , \quad (45)$$

if both of these windows are situated in the central rapidity region, where the translation invariance in rapidity takes place.

In other case when the FB windows are $\delta\varphi_F = \delta\varphi_B = 2\pi$ in azimuth and small in rapidity, we have

$$b_{rob} = \frac{C_2(y_F, y_B)}{\sqrt{C_2(y_F, y_F) C_2(y_B, y_B)}} , \quad (46)$$

where $C_2(y_F, y_B)$ is defined by (27). In the central rapidity region $C_2(y_F, y_B) = C_2(y_{FB})$ and we have

$$b_{rob} = \frac{C_2(y_{FB})}{C_2(0)} . \quad (47)$$

We see that in contrast with b the b_{rob} as the b_{mod} has a finite limit at small acceptances of windows.

Note that the definition (43) is closely connected with so-called robust variance [23, 24, 25]:

$$R_n = \frac{D_n - \langle n \rangle}{\langle n \rangle^2} . \quad (48)$$

By (40) and (43) we have

$$b_{rob} = \frac{b_{mod}}{\sqrt{R_{n_F} R_{n_B}}} . \quad (49)$$

Emphasize that the traditionally defined (2) correlation coefficient b is also proportional to the two-particle correlation function $C_2(y_F, y_B; \varphi_{FB})$ (see (33)), but the proportionality factor depends on the width of windows and goes to zero at $\delta_F = \delta_B \rightarrow 0$.

5 Untriggered di-hadron correlation

The following alternative definition of two-particle correlation function is also in use [26, 27]:

$$C = S/B - 1 , \quad (50)$$

where

$$S = \frac{d^2 N}{d\Delta y d\Delta\varphi} . \quad (51)$$

Here $\Delta y = y_1 - y_2$ and $\Delta\varphi = \varphi_1 - \varphi_2$ are the distances between two particles in rapidity and in azimuth, and one takes into account all possible pair combinations of particles produced in given event in some rapidity interval (Y_1, Y_2) . The B is the same but for the case of uncorrelated particle production.

At this definition in contrast with (17) one implies from the very beginning that the translation invariance in rapidity takes place. Namely, that for any y_1 and y_2 belonging to the interval (Y_1, Y_2) the result depends only on $\Delta y = y_1 - y_2$. (All the pairs with the same value of difference $y_1 - y_2$ contribute to the same bin of the multiplicity distribution, irrespective of the value of $(y_1 + y_2)/2$, see also the discussion in [18].) This assumption is reasonable only in the central rapidity region at high energies. It means that we suppose that in the interval (Y_1, Y_2) :

$$\rho_1(y) = \rho_0 , \quad \rho_2(y_1, y_2; \varphi) = \rho_2(y_1 - y_2; \varphi) \quad (52)$$

(see formula (34)).

In this case we have for the enumerator of (50):

$$S(\Delta y, \Delta\varphi) = \int_{Y_1}^{Y_2} dy_1 dy_2 \rho_2(y_1 - y_2; \Delta\varphi) \delta(y_1 - y_2 - \Delta y) \quad (53)$$

or in the case of commonly used symmetric interval $(-Y/2, Y/2)$:

$$S(\Delta y, \Delta\varphi) = \rho_2(\Delta y; \Delta\varphi) t_Y(\Delta y) \quad (54)$$

where the $t_Y(\Delta y)$ is a "triangular" weight function (102), defined in Appendix A (see Fig.2).

In the denominator of (50) we should replace the $\rho_2(y_1, y_2; \Delta\varphi)$ by the product $\rho_1(y_1)\rho_1(y_2)$, which due to the translation invariance in rapidity reduces simply to ρ_0^2 . Then

$$B(\Delta y, \Delta\varphi) = \rho_0^2 t_Y(\Delta y) . \quad (55)$$

Substituting into (50) we get

$$C(\Delta y, \Delta\varphi) = \frac{\rho_2(\Delta y; \Delta\varphi)}{\rho_0^2} - 1 = C_2(\Delta y, \Delta\varphi) , \quad (56)$$

where we have taken into account (17) and (34). We see that if the translation invariance in rapidity takes place within the interval (Y_1, Y_2) , then the definition (50) is equivalent to the standard one (17) (see meanwhile the remark in the end of the section 7).

The drawback of this approach is that it supposes from the very beginning the translation invariance and hence can't be applied for an investigation of the multiplicity correlation at large rapidity distances, where the translation invariance in rapidity (34) is not valid. At that by (33), (41) and (44) we see that the approaches based on the analysis of the standard (2) or modified (40), (43) the FB correlation coefficients with two remote windows of small acceptance in rapidity and azimuth enable to measure the correlation strength $C_2(y_1, y_2; \varphi_1 - \varphi_2)$ also in this case.

6 The model. Pair correlation function of single string

We now calculate the FB correlations in windows separated in rapidity and azimuth using the simple two stage model [9, 10, 20], inspired by a string picture of hadronic interactions. In this model we suggest that at the initial stage of interaction some number N of strings are formed, which fluctuates event-by-event with some variance $D_N = \langle N^2 \rangle - \langle N \rangle^2$ or scaled variance

$$\omega_N = D_N / \langle N \rangle . \quad (57)$$

Note that the fluctuation in the number of strings in pp and especially in AA collisions [28] is not poissonian and hence $\omega_N \neq 1$. Its value depends on the collision energy.

At next stage we consider these strings as identical independent emitters of observed charge particles. In previous note [21] we have considered only the so-called long-range (LR) part of the correlation, originating from the fluctuation in the number of strings. In the present paper we also take into account the short-range (SR) contribution, originating from the correlation between particles produced by a single string.

To characterize the last property of the string we introduce, similarly to the consideration in the section 3, the two-particle correlation function for charged particles produced from a decay of a single string $\Lambda(y_1, y_2; \varphi_1 - \varphi_2)$. For this purpose we at first introduce the $\lambda_1(y, \varphi)$ and $\lambda_2(y_1, \varphi_1; y_2, \varphi_2)$ - the one- and two-particle densities of charge particles produced by one string. Then for given acceptance interval $\delta y_F \delta \varphi_F$:

$$\int_{\delta y_F \delta \varphi_F} dy d\varphi \lambda_1(y, \varphi) = \langle \mu_F \rangle , \quad (58)$$

$$\int_{\delta y_F \delta \varphi_F} dy_1 d\varphi_1 \int_{\delta y_F \delta \varphi_F} dy_2 d\varphi_2 \lambda_2(y_1, \varphi_1; y_2, \varphi_2) = \langle \mu_F (\mu_F - 1) \rangle$$

and the same for backward window $\delta y_B \delta \varphi_B$. Whereas

$$\int_{\delta y_F \delta \varphi_F} dy_1 d\varphi_1 \int_{\delta y_B \delta \varphi_B} dy_2 d\varphi_2 \lambda_2(y_1, \varphi_1; y_2, \varphi_2) = \langle \mu_B \mu_F \rangle . \quad (59)$$

The $\langle \mu_F \rangle$ and $\langle \mu_B \rangle$ are the average multiplicities produced by one string in the forward $\delta y_F \delta \varphi_F$ and backward $\delta y_B \delta \varphi_B$ windows. In general case due to the rotation invariance in azimuth one has

$$\lambda_1(y, \varphi) = \lambda_1(y) / 2\pi , \quad \lambda_2(y_1, \varphi_1; y_2, \varphi_2) = \lambda_2(y_1, y_2; \varphi_1 - \varphi_2) / (2\pi)^2 . \quad (60)$$

Similarly (17) we can also introduce two-particle correlation function for charged particles produced from a decay of a single string by a standard way:

$$\Lambda(y_1, y_2; \varphi_1 - \varphi_2) = \frac{\lambda_2(y_1, y_2; \varphi_1 - \varphi_2)}{\lambda_1(y_1)\lambda_1(y_2)} - 1 . \quad (61)$$

By (58)–(61) we have

$$\langle \mu_B \mu_F \rangle - \langle \mu_B \rangle \langle \mu_F \rangle = J_{FB} , \quad (62)$$

$$D_{\mu_F} = \langle \mu_F \rangle + J_{FF} , \quad (63)$$

where

$$\langle \mu_F \rangle = \frac{\delta\varphi_F}{2\pi} \int_{\delta y_F} dy \lambda_1(y) , \quad (64)$$

$$J_{FB} = (2\pi)^{-2} \int_{\delta y_F \delta\varphi_F} dy_1 d\varphi_1 \int_{\delta y_B \delta\varphi_B} dy_2 d\varphi_2 \lambda_1(y_1) \lambda_1(y_2) \Lambda(y_1, y_2; \varphi_1 - \varphi_2) , \quad (65)$$

$$J_{FF} = (2\pi)^{-2} \int_{\delta y_F \delta\varphi_F} dy_1 d\varphi_1 \int_{\delta y_F \delta\varphi_F} dy_2 d\varphi_2 \lambda_1(y_1) \lambda_1(y_2) \Lambda(y_1, y_2; \varphi_1 - \varphi_2) . \quad (66)$$

Further simplification of the integrals are described in Appendix A.

By (63) we see that the presence of SR correlation turns the string into non-poissonian emitter.

For small windows in which one can consider $\Lambda(y_1, y_2; \varphi_1 - \varphi_2)$ and $\lambda_1(y)$ to be constant

$$\langle \mu_F \rangle = \lambda_1(y_F) \delta_F , \quad \langle \mu_B \rangle = \lambda_1(y_B) \delta_B , \quad (67)$$

$$J_{FB} = \delta_F \delta_B \lambda_1(y_F) \lambda_1(y_B) \Lambda(y_F, y_B; \varphi_{FB}) , \quad (68)$$

$$J_{FF} = \delta_F^2 \lambda_1^2(y_F) \Lambda(y_F, y_F; 0) , \quad (69)$$

where we have used our short notations δ_F and δ_B (8) for the window acceptances.

If the both small windows situated in the central rapidity region, where each string contributes to the particle production in the whole rapidity region, then due to the translation invariance in rapidity

$$\lambda_1(y) = \mu_0 , \quad \Lambda(y_1, y_2; \varphi) = \Lambda(y_1 - y_2; \varphi) \quad (70)$$

and the formulae (67)–(69) take the form

$$\langle \mu_F \rangle = \mu_0 \delta_F , \quad \langle \mu_B \rangle = \mu_0 \delta_B , \quad (71)$$

$$J_{FB} = \delta_F \delta_B \mu_0^2 \Lambda(y_{FB}; \varphi_{FB}) , \quad (72)$$

$$J_{FF} = \delta_F^2 \mu_0^2 \Lambda(0; 0) . \quad (73)$$

Recall that y_{FB} and φ_{FB} are the distances between the centers of forward and backward windows in rapidity and azimuth (8).

Note that in the case of large windows situated in the central rapidity region along with (71) one must use the formulae (62) and (63) with J_{FB} and J_{FF} given by the following expressions

$$J_{FB} = \mu_0^2 (2\pi)^{-2} \int_{\delta y_F \delta\varphi_F} dy_1 d\varphi_1 \int_{\delta y_B \delta\varphi_B} dy_2 d\varphi_2 \Lambda(y_1 - y_2; \varphi_1 - \varphi_2) , \quad (74)$$

$$J_{FF} = \mu_0^2 (2\pi)^{-2} \int_{\delta y_F \delta\varphi_F} dy_1 d\varphi_1 \int_{\delta y_F \delta\varphi_F} dy_2 d\varphi_2 \Lambda(y_1 - y_2; \varphi_1 - \varphi_2) \quad (75)$$

(see Appendix A for further simplifications).

7 The model. Resulting correlation strength.

In a general case in the model with N independent identical emitters [23]:

$$\rho_1^N(y) = N\lambda_1(y) , \quad (76)$$

$$\rho_2^N(y_1, y_2; \varphi) = N\lambda_2(y_1, y_2; \varphi) + N(N-1)\lambda_1(y_1)\lambda_1(y_2) . \quad (77)$$

Then the one- and two-particle densities of charge particles (11) are given by

$$\rho_1(y) = \langle \rho_1^N(y) \rangle = \langle N \rangle \lambda_1(y_1) , \quad (78)$$

$$\rho_2(y_1, y_2; \varphi) = \langle \rho_2^N(y_1, y_2; \varphi) \rangle = \langle N \rangle [\lambda_2(y_1, y_2; \varphi) - \lambda_1(y_1)\lambda_1(y_2)] + \langle N^2 \rangle \lambda_1(y_1)\lambda_1(y_2) , \quad (79)$$

and

$$\rho_2(y_1, y_2; \varphi) - \rho_1(y_1)\rho_1(y_2) = \langle N \rangle [(\lambda_2(y_1, y_2; \varphi) - \lambda_1(y_1)\lambda_1(y_2))] + D_N \lambda_1(y_1)\lambda_1(y_2) , \quad (80)$$

where D_N is the event-by-event variance $D_N = \langle N^2 \rangle - \langle N \rangle^2$ of the number of emitters. As a result we have the following expression of the two-particle correlation function $C_2(y_F, y_B; \varphi_{FB})$ (17) through the pair correlation function of a single string $\Lambda(y_F, y_B; \varphi_{FB})$ (61):

$$C_2(y_1, y_2; \varphi) = \frac{\omega_N + \Lambda(y_1, y_2; \varphi)}{\langle N \rangle} , \quad (81)$$

where ω_N is the event-by-event scaled variance $\omega_N = D_N / \langle N \rangle$ of the number of emitters (57).

In the case of FB windows which are small both in rapidity and in azimuth the (81) by (33) leads to the following formula for the FB correlation coefficient (4):

$$b_{rel} = \frac{\langle n_F \rangle}{\langle n_B \rangle} b = \frac{\langle n_F \rangle [\omega_N + \Lambda(y_F, y_B; \varphi_{FB})] / \langle N \rangle}{1 + \langle n_F \rangle [\omega_N + \Lambda(y_F, y_F; 0)] / \langle N \rangle} . \quad (82)$$

If these small FB windows are situated in the central rapidity region, where the translation invariance in rapidity takes place, then the $\Lambda(y_F, y_B; \varphi_{FB})$ will depend only on the difference $y_{FB} = y_F - y_B$ of rapidities and the (82) takes the following form

$$b_{rel} = \frac{\delta_F}{\delta_B} b = \frac{\delta_F \mu_0 [\omega_N + \Lambda(y_{FB}; \varphi_{FB})]}{1 + \delta_F \mu_0 [\omega_N + \Lambda(0; 0)]} , \quad (83)$$

where μ_0 is the average rapidity density of the charged particles produced by one string. Note that in this case the basic formula (81) can be obtained also by an alternative way in the framework of the two stage model [9, 10, 20] (see Appendix B).

One can present the result for the FB correlation coefficient (83) as the sum of two terms $b_{rel} = b_{rel}^{LR} + b_{rel}^{SR}$, where:

$$b_{rel}^{LR} = \frac{\delta_F \mu_0 \omega_N}{1 + \delta_F \mu_0 [\omega_N + \Lambda(0; 0)]} , \quad (84)$$

and

$$b_{rel}^{SR} = \frac{\delta_F \mu_0}{1 + \delta_F \mu_0 [\omega_N + \Lambda(0; 0)]} \Lambda(y_{FB}; \varphi_{FB}) . \quad (85)$$

The first term depends on the acceptance δ_F of the forward window, but doesn't depend on the distance between the centers of forward and backward windows in rapidity y_{FB} and in azimuth φ_{FB} , which justifies the name of this contribution as the long range (LR) one. This contribution reveals itself as the common "plateau" when one plots the value of the

FB correlation coefficient b as a function of y_{FB} and φ_{FB} . The level of this "plateau" is determined by the event-by-event fluctuation of the number of the strings (emitters) N and can be used for the evaluation of the extent of this fluctuation. Note that at any fixed number of emitters there will be no such contribution, as $\omega_N \equiv D_N/\langle N \rangle = 0$.

The second term is proportional to the pair correlation function $\Lambda(y_{FB}; \varphi_{FB})$ of a single string with some common factor depending on acceptance. In the plot of the FB correlation coefficient b as a function of y_{FB} and φ_{FB} this contribution manifests itself as some peaks above the level of the common "plateau" (see Fig.1 below). This justified the name of this contribution as the short range (SR) one. This different behavior enables separate the contribution of two mechanisms.

We would like to emphasize that if the pair correlation function of a single string is equal to zero: $\Lambda(y_{FB}; \varphi_{FB}) = 0$, we still have nonzero FB correlation:

$$b_{rel}^{\Lambda=0} = \frac{\delta_F \mu_0 \omega_N}{1 + \delta_F \mu_0 \omega_N} , \quad (86)$$

which characterizes the event-by-event fluctuation of the number of strings N . Note also that at $\Lambda(y_{FB}; \varphi_{FB}) = 0$ by (63) and (66) the strings become a poissonian emitters and the answer (86) coincides with our result, obtained in [10, 20, 21] for this case.

For the alternative observables b_{mod} (40) and b_{rob} (43), introduced in section 4, in the framework of the model we have:

$$b_{mod} = \frac{\omega_N + \Lambda(y_F, y_B; \varphi_{FB})}{\langle N \rangle} , \quad (87)$$

$$b_{rob} = \frac{\omega_N + \Lambda(y_F, y_B; \varphi_{FB})}{\omega_N + \Lambda(y_F, y_F; 0)} . \quad (88)$$

We see that they do not depend on the windows acceptances δ_F , δ_B and have simple connection with the pair correlation function of a single string $\Lambda(y_F, y_B; \varphi_{FB})$.

In the case of large acceptance windows, within which one can't consider $\Lambda(y_1, y_2; \varphi)$ to be constant, by (18)–(23) and (76)–(81) the formulae (82), (87) and (88) still can be used but with the following substitution:

$$\Lambda(y_F, y_B; \varphi_{FB}) \rightarrow \frac{1}{\langle \mu_F \rangle \langle \mu_B \rangle} \int_{\delta y_F \delta \varphi_F} dy_1 d\varphi_1 \int_{\delta y_B \delta \varphi_B} dy_2 d\varphi_2 \lambda_1(y_1) \lambda_1(y_2) \Lambda(y_1, y_2; \varphi_1 - \varphi_2) , \quad (89)$$

where $\langle \mu_F \rangle$ and $\langle \mu_B \rangle$ are the mean multiplicities, produced in the forward and backward windows by a single emitter (64):

$$\langle \mu_F \rangle = \frac{\delta \varphi_F}{2\pi} \int_{\delta y_F} dy \lambda_1(y) , \quad \langle \mu_B \rangle = \frac{\delta \varphi_F}{2\pi} \int_{\delta y_F} dy \lambda_1(y) . \quad (90)$$

In the central region, due to translation invariance in rapidity, this substitution for large acceptance windows looks more simple:

$$\Lambda(y_{FB}; \varphi_{FB}) \rightarrow (\delta y_F \delta \varphi_F \delta y_B \delta \varphi_B)^{-1} \int_{\delta y_F \delta \varphi_F} dy_1 d\varphi_1 \int_{\delta y_B \delta \varphi_B} dy_2 d\varphi_2 \Lambda(y_1 - y_2; \varphi_1 - \varphi_2) , \quad (91)$$

$$\Lambda(0; 0) \rightarrow (\delta y_F \delta \varphi_F)^{-2} \int_{\delta y_F \delta \varphi_F} dy_1 d\varphi_1 \int_{\delta y_F \delta \varphi_F} dy_2 d\varphi_2 \Lambda(y_1 - y_2; \varphi_1 - \varphi_2) . \quad (92)$$

Further simplifications of the integrals are discussed in Appendix A.

For example, in the important case of symmetric ($\delta y_F = \delta y_B = \delta y$) FB windows, situated in the central rapidity region and separated only in rapidity (i.e. at $\delta\varphi_F = \delta\varphi_B = 2\pi$ acceptance in azimuth), by (101), (103) and (105) the formulae (91) and (92) transform to

$$\Lambda(y_{FB}; \varphi_{FB}) \rightarrow \frac{1}{\delta y} \int_{-\delta y}^{\delta y} dy \Lambda(y + y_{FB}) t_{\delta y}(y) , \quad (93)$$

$$\Lambda(0; 0) \rightarrow \frac{2}{\delta y} \int_0^{\delta y} dy \Lambda(y)(\delta y - y) = 2 \int_0^{\delta y} \Lambda(y) dy - \frac{2}{\delta y} \int_0^{\delta y} y \Lambda(y) dy , \quad (94)$$

where $t_{\delta y}(y)$ is a "triangular" weight function (102) (see Fig.2) and

$$\Lambda(y) = \frac{1}{\pi} \int_0^\pi d\varphi \Lambda(y; \varphi) . \quad (95)$$

We have used also that

$$\Lambda(-y; \varphi) = \Lambda(y; \varphi) , \quad \Lambda(y; -\varphi) = \Lambda(y; \varphi) , \quad \Lambda(y; \varphi + 2\pi k) = \Lambda(y; \varphi) . \quad (96)$$

In a conclusion of the section we note that if one uses the so-called di-hadron correlation approach, described above in section 5, for the experimental determination of the two-particle correlation function $C(\Delta y, \Delta\varphi)$ (50) the result will depend on the details of "track and/or event mixing" used in that approach for the determination of B by the imitation of the "uncorrelated" particle production.

If, as it was supposed in section 5 (55):

$$\begin{aligned} B(\Delta y, \Delta\varphi) &= \int_{-Y/2}^{Y/2} dy_1 dy_2 \rho_1(y_1) \rho_1(y_2) \delta(y_1 - y_2 - \Delta y) = \\ &= \int_{-Y/2}^{Y/2} dy_1 dy_2 \langle \rho_1^N(y_1) \rangle \langle \rho_1^N(y_2) \rangle \delta(y_1 - y_2 - \Delta y) = \rho_0^2 t_Y(\Delta y) = \langle N \rangle^2 \mu_0^2 t_Y(\Delta y) , \end{aligned} \quad (97)$$

then by (50), (54) and (79) with $\lambda_1(y) = \mu_0$ we get

$$C(\Delta y, \Delta\varphi) = \frac{\omega_N + \Lambda(\Delta y, \Delta\varphi)}{\langle N \rangle} = C_2(\Delta y, \Delta\varphi) , \quad (98)$$

which under the assumption of the translation invariance in the central rapidity region corresponds to the two-particle correlation function $C_2(y_1, y_2; \varphi)$, defined by the standard way (17), (compare with (81)).

But if instead of (97) one has

$$B(\Delta y, \Delta\varphi) = \int_{-Y/2}^{Y/2} dy_1 dy_2 \langle \rho_1^N(y_1) \rho_1^N(y_2) \rangle \delta(y_1 - y_2 - \Delta y) = \langle N^2 \rangle \mu_0^2 t_Y(\Delta y) , \quad (99)$$

as it frequently takes place in a di-hadron data analysis, then instead of (98) by (50), (54) and (79) we get

$$C(\Delta y, \Delta\varphi) = \frac{\langle N \rangle}{\langle N^2 \rangle} \Lambda(\Delta y, \Delta\varphi) , \quad (100)$$

which does not correspond to the standard two-particle correlation function $C_2(y_1, y_2; \varphi)$, defined by (17). Compare (100) with (98) we see that in this case the resulting $C(\Delta y, \Delta\varphi)$ does not have an additional contribution reflecting the event-by-event fluctuation in the number of emitters. It depends only on the pair correlation function of a single string $\Lambda(\Delta y, \Delta\varphi)$ and, therefore, is equal to zero in the absence of the pair correlation from one string.

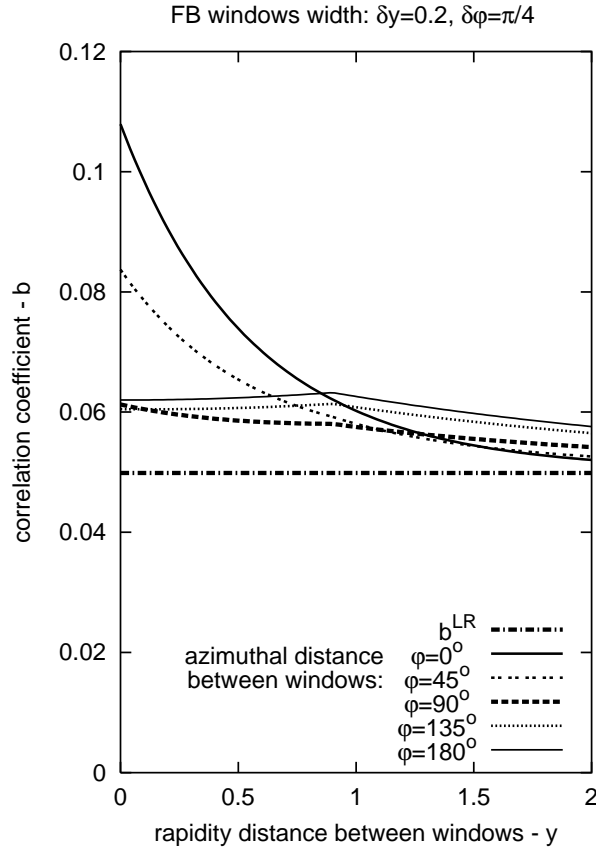


Figure 1: The forward-backward (FB) correlation coefficient b (5) for symmetric windows of small acceptance $\delta y_F = \delta y_B = \delta y = 0.2$ and $\delta \varphi_F = \delta \varphi_B = \delta \varphi = \pi/4$, calculated with taking into account the long-range (LR) and short-range (SR) contributions (83)–(85), at different values of a distance between the centers of the windows in azimuth $\varphi \equiv \varphi_{FB}$ as a function of a distance between the centers of the windows in rapidity $y \equiv y_{FB}$.

8 Rapidity-azimuth dependence of the FB multiplicity correlation strength

We analyze at first the FB multiplicity correlation strength in the most simple case of windows, which are small both in azimuth and in rapidity and situated in the central region, what is described by the formulae (83)–(85).

As an illustration, the behavior of the correlation coefficient in the case of small FB windows, which are $\delta \varphi_F = \delta \varphi_B = \delta \varphi = \pi/4$ in azimuth and $\delta y_F = \delta y_B = \delta y = 0.2$ in rapidity, with $\Lambda(y; \varphi)$ expected from the Schwinger mechanism of a string break up are shown in Fig.1 [29]. (Note that in this case $b_{rel} = b$.) There are the large narrow nearside peak at $y_{FB} = 0$ and $\varphi_{FB} = 0$, the some away-side structure at $\varphi_{FB} = \pi$, which is smaller in amplitude and wider in rapidity, and the common "plateau" (pedestal) corresponding to the contribution of LR correlation, originating from the fluctuation in the number of emitting sources.

Using the experimental data on the FB multiplicity correlation strength with small windows in azimuth and rapidity situated at different rapidity y_{FB} and azimuth φ_{FB} distances from each other, one can fix all parameters entering the SR (the parameters of the $\Lambda(y; \varphi)$ function) and the LR (the parameter ω_N) contributions. (Note that only the products $\mu_0 \Lambda(y; \varphi)$

and $\mu_0\omega_N$ are entering in the resulting formula (83).)

With the parameters, fixed in this way by the data on the FB correlation coefficient b with small acceptance windows, one can calculate (without any additional free parameters) using formulae (91) and (92) the values of the FB correlation coefficient b for large acceptance windows, within which one cannot consider the $\Lambda(y, \varphi)$ to be constant.

For example, one can calculate the values of the FB correlation coefficient b in the practically important case of symmetric windows separated only in rapidity, i.e. for 2π -windows in azimuth. In this case the resulting formula (83) with the substitutions (93) and (94) gives the dependencies of the FB correlation coefficient b on the width (δy) of windows and on the rapidity gap ($y_{gap} \equiv y_{FB} - \delta y$) (10) between them.

9 Conclusions

We have analyzed and compared the different definitions of the multiplicity correlation coefficient. We see that the traditional definitions (2), (4) and (5) of the FB correlation coefficient lead to its strong dependence on the acceptance of the windows, with the correlation coefficient going to zero with the acceptance. Hence, the results obtained for the windows of different width can't be compared directly. In this connection we propose suitable observables (40) and (43) for the FB correlation studies, which values have some nonzero limit when the acceptance go to zero.

We have extended the definitions of the multiplicity correlation coefficient to the case of the windows separated both in rapidity and in azimuth. We have showed that this enables to separate the contribution, arising due to the event-by-event fluctuation in the number of sources, and the contribution, originating from the pair correlation function of a single source. The last can reflect the very different physical processes such as the details of string break up, the formation and decay of clusters, resonances or minijets during the string fragmentation.

In mid-rapidity region the first contribution, originating from the fluctuation in the number of emitters, doesn't depend on the distance between the windows in rapidity and azimuth, which justifies the name of this contribution as the long range (LR) one. This LR contribution is proportional to the scaled event-by-event variance of the number of emitters ω_N .

The analysis of this contribution enables to obtain the important quantitative physical information on the magnitude of this fluctuation in the processes under consideration. At that the strong non-linear dependence of the LR contribution on the window acceptances, taking place in the case of the traditionally defined correlation coefficient (2) and (4), is fully specified (84).

The second contribution, originating from the correlation between multiplicities produced by a single emitter, depends on the distances between the centers of the backward and forward windows in rapidity y_{FB} and in azimuth φ_{FB} . Its value (85) is proportional to the pair correlation function of a single source $\Lambda(y_{FB}, \varphi_{FB})$ and decreases at large separations, which justifies the name of this contribution as the short range (SR) one. The dependence of the SR contribution on y_{FB} and φ_{FB} can be retrieved from the experimental data on the multiplicity correlation with the FB windows, which are small both in rapidity and in azimuth.

We also see that the presence of such SR correlation along with the influence on the FB multiplicity correlation inevitably turns a string into a non-poissonian emitter.

We trace the connection of the FB correlation coefficient with the two-particle correlation function $C_2(y_F, y_B; \varphi_{FB})$ and show that the standardly defined two-particle correlation function in general case is the sum of the contributions of the two above (LR and SR) mechanisms (see equation (81)).

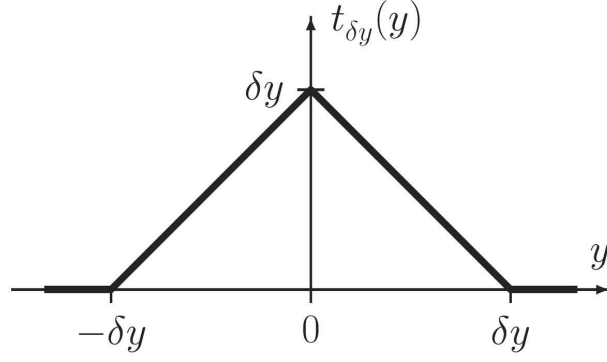


Figure 2: The "triangular" weight function arising due to phase space at integration over non-periodical FB windows (see Appendix A for details).

We also discuss the so-called di-hadron correlation approach $C(y, \varphi)$ (50) to the investigation of the multiplicity correlation function. The obvious disadvantage of this approach is that it supposes from the very beginning the translation invariance in rapidity and hence can't be applied for an investigation of the multiplicity correlation at large rapidity distances, where this invariance is not valid. At that we show that the approaches based on the analysis of the standard (2) or modified (40), (43) the FB correlation coefficients with two remote windows of small acceptance in rapidity and azimuth enable to measure the correlation strength $C_2(y_F, y_B; \varphi_{FB})$ also in this case.

Even in the mid-rapidity region, where the application of the di-hadron correlation approach is justified, the results obtained by this approach depend on the details of "track and/or event mixing" used in this approach for the imitation of the "uncorrelated" particle production. As it was shown in the end of the section 7, the obtained correlation coefficient $C(y, \varphi)$ can be equal to the standardly defined two-particle correlation function $C_2(y, \varphi)$ (98) and contain the sum of LR and SR contributions, or it can be proportional only to the SR contribution (100) in the dependence on details of the procedure applied. In the last case it loses the common "pedestal", the height of which is proportional to the scaled event-by-event variance of the number of emitters ω_N .

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Appendix A. Calculation of integrals over rapidity and azimuth windows.

For symmetric rapidity windows $\delta y_B = \delta y_F = \delta y$ with the distance $y_{FB} = y_F - y_B$ between their centers one has

$$\int_{\delta y_F} dy_1 \int_{\delta y_B} dy_2 f(|y_1 - y_2|) = \int_{-\delta y}^{\delta y} dy f(|y_{FB} + y|) t_{\delta y}(y) , \quad (101)$$

where $t_{\delta y}(y)$ is a "triangular" weight function (see Fig.2):

$$t_{\delta y}(y) = [\theta(-y)(\delta y + y) + \theta(y)(\delta y - y)] \theta(\delta y - |y|) . \quad (102)$$

The formula (101) is valid for any distance between the centers of windows, in particular for coinciding windows. In the last case $y_{FB} = 0$ and we have

$$\int_{\delta y_F} dy_1 \int_{\delta y_F} dy_2 f(|y_1 - y_2|) = \int_{-\delta y}^{\delta y} dy f(|y|) t_{\delta y}(y) = 2 \int_0^{\delta y} dy f(|y|) (\delta y - y) \quad (103)$$

The same formula

$$\int_{\delta \varphi_F} d\varphi_1 \int_{\delta \varphi_B} d\varphi_2 f(|\varphi_1 - \varphi_2|) = \int_{-\delta \varphi}^{\delta \varphi} d\varphi f(|\varphi_{FB} + \varphi|) t_{\delta \varphi}(\varphi) \quad (104)$$

is valid for the integration over azimuthal windows, but in this case one has also to take into account the periodicity: $f(|\varphi|) = f(|\varphi + 2\pi k|)$. The last leads to significant simplification of the formula (104) in the case of windows of full 2π acceptance in azimuth:

$$\int_{-2\pi}^{2\pi} d\varphi f(|\varphi_{FB} + \varphi|) t_{2\pi}(\varphi) = 2\pi \int_0^{2\pi} d\varphi f(|\varphi|) = 4\pi \int_0^{\pi} d\varphi f(|\varphi|) . \quad (105)$$

So for large symmetric windows in the central rapidity region by (101)–(103) the formulae (91) and (92) in general case can be written in the following form

$$\Lambda(y_{FB}; \varphi_{FB}) \rightarrow (\delta y \delta \varphi)^{-2} \int_{-\delta y}^{\delta y} dy \int_{-\delta \varphi}^{\delta \varphi} d\varphi \Lambda(y_{FB} + y; \varphi_{FB} + \varphi) t_{\delta y}(y) t_{\delta \varphi}(\varphi) , \quad (106)$$

$$\Lambda(0; 0) \rightarrow 4(\delta y \delta \varphi)^{-2} \int_0^{\delta y} dy \int_0^{\delta \varphi} d\varphi \Lambda(y; \varphi) (\delta y - y) (\delta \varphi - \varphi) . \quad (107)$$

The δy and $\delta \varphi$ are the width of the observation windows in rapidity and in azimuth, and the y_{FB} and φ_{FB} are the corresponding distances between their centers. We imply that $\Lambda(y; \varphi)$ satisfies the conditions (96). The same simplifications take place for the integrals I_{FB} (38) and I_{FF} (39) in section 3.

Appendix B. Connection of resulting correlator and variance with the ones for single emitter

In the framework of the two stage model [9, 10, 20] in the case of the FB windows in the central rapidity region one can express the observable correlator $\langle n_F n_B \rangle - \langle n_F \rangle \langle n_B \rangle$ and the variance D_{n_F} through the correlator $\langle \mu_F \mu_B \rangle - \langle \mu_F \rangle \langle \mu_B \rangle$ and the variance D_{μ_F} for one emitter:

$$\langle n_F n_B \rangle - \langle n_F \rangle \langle n_B \rangle = \langle N \rangle (\langle \mu_F \mu_B \rangle - \langle \mu_F \rangle \langle \mu_B \rangle) + D_N \langle \mu_F \rangle \langle \mu_B \rangle , \quad (108)$$

$$D_{n_F} = \langle N \rangle D_{\mu_F} + D_N \langle \mu_F \rangle^2 , \quad (109)$$

where $D_N = \langle N^2 \rangle - \langle N \rangle^2$ and $\langle N \rangle$ are the event-by-event variance and the mean number of emitters (see [21, 29] for derivation). These formulae are obtained for the windows situated in the central rapidity region, where each string contributes to the particle production in the whole rapidity range and hence the translation invariance in rapidity takes place. In this region by (35) and (71) we have also

$$\langle n_F \rangle = \delta_F \rho_0 , \quad \langle \mu_F \rangle = \delta_F \mu_0 , \quad \langle n_B \rangle = \delta_B \rho_0 , \quad \langle \mu_B \rangle = \delta_B \mu_0 , \quad \rho_0 = \langle N \rangle \mu_0 . \quad (110)$$

Recall that μ_0 is the average rapidity density of the charged particles produced by one string.

In the case of FB windows which are small in rapidity and azimuth by (18) and (30) we have

$$C_2(y_F, y_B; \varphi_{FB}) = \frac{\langle n_F n_B \rangle - \langle n_F \rangle \langle n_B \rangle}{\langle n_F \rangle \langle n_B \rangle}. \quad (111)$$

Similarly by (62) and (72) we have the same for the pair correlation function $\Lambda(y_B, y_F; \varphi_{FB})$ of a single string:

$$\Lambda(y_B, y_F; \varphi_{FB}) = \frac{\langle \mu_F \mu_B \rangle - \langle \mu_B \rangle \langle \mu_F \rangle}{\langle \mu_B \rangle \langle \mu_F \rangle}. \quad (112)$$

Then combining the formulae (108)–(112) we again get the formula (81) of the text in this particular case.

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